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BIOGRAPHY.

DE MORGAN.

BY GEORGE BRUCE HALSTED.

AUGUSTUS DE MORGAN is fortunately a well-known character. The world has an excellent sketch of him by his pupil Jevons in the Encyclopædia Britannica, from which I copy literally some of the preliminary biographic data here given. If I am able to add anything specifically new to this part of the paper, it is from a life-long study of his works, and many conversations about him with the great Sylvester, who knew him very intimately.

Augustus De Morgan was born in June, 1806, in India. On his mother's side he was descended from James Dodson, F. R. S., author of the Anti-Logarithmic Canon and other mathematical works of merit, and a friend of De Moivre. Our Augustus lost one eye in his early infancy, and this prevented his joining in the usual games. He read algebra "like a novel," and pricked out equations on the school pew instead of listening to the sermons.

When 16 years old he entered Trinity College, Cambridge, and studied his mathematics partly under the tuition of Airy, making many friends, including his teachers Whewell and Peacock. In 1825 he gained a Trinity Scholarship.

But De Morgan's attention was by no means confined to mathematics, and his love of wide reading somewhat interfered with his success in the mathematical tripos, in which he took fourth place in 1827, before he had completed his 21st year.

He was prevented from taking his M. A. degree, or from obtaining a fel-

A. De Morgan

lowship, by his conscientious objection to signing the theological tests then required from masters of arts and Fellows at Cambridge. Jevons says, "a strong repugnance to any sectarian restraint upon the freedom of opinion was one of De Morgan's most marked characteristics throughout life."

A career in his own university being thus closed against him, on the twenty-third of February, 1828, he was elected Professor of Mathematics in University College, London, and began lecturing at the early age of 22 years.

But the regents of this College claimed the right of dismissing a professor without assigning reasons, and acted upon this principle in dismissing the Professor of Anatomy. Immediately De Morgan resigned in protest. After the regulations were changed he was invited to resume his chair, was reappointed, and served for the next 30 years. His dislike to honorary titles led him to refuse the offer of LL. D. from the University of Edinburgh.

In 1866 a discussion arose as to the true interpretation of the principle of religious neutrality avowedly adopted by the College. De Morgan held that any consideration of a candidate's ecclesiastical position or creed or lack of creed was inconsistent with the principle. In protest against a violation of this principle he resigned a second time in a letter dated November 10th, 1866. He was always remarkably free from any touch of sordid self-interest.

As a teacher, De Morgan was particularly gifted. A voluminous writer on mathematics, he contributed essentially to those expansions of the fundamental concepts which have rendered possible the new algebras, such as Quaternions, and have generalized the whole idea of a mathematical algorithm, until now an American produces a thesis entitled "Mathematics the Science of Algorithms."

His logical work alone would give De Morgan lasting fame. Here he stands alongside his immortal contemporary, Boole. The eternally memorable year in the history of Logic was 1847, in which George Boole issued "The Mathematical Analysis of Logic, being an Essay toward a Calculus of Deductive Reasoning," von Staudt published his "Geometrie der Lage," a mathematics utterly freed from any idea of quantity, a mathematics strictly qualitative, and De Morgan printed his fundamental treatise, called "Formal Logic; or, the Calculus of Inference, Necessary and Probable."

The great memoirs produced in 1850, 1858, 1860, 1863 are preserved, if buried, in the inaccessible "Cambridge Philosophical Transactions." De Morgan's great combination of logical with mathematical learning, and his prominent position in London, the great metropolis, made him the man to whom resorted all the Circle-Squarers, Angle-Trisectors, Perpetual-Motionists, Triangle-Angle-sumers. Adding this curious experience to his great bibliographic knowledge of what had been attempted in that way in the past, he formed a large book called "A Budget of Paradoxes," which is one of the most interesting treatises ever written on what may be called extended fallacies. This charming book has steadily advanced in market price until I find it cited in Macmillan's Catalogue No. 245 (1086) at fifty shillings per copy.

It was De Morgan who first gave a thorough treatment of contrary, negative, or contradictory terms, though in just the sense I have heard babies use them while learning our language. But remember it is Darwinism that has since taught the world to learn at the feet of babes. Bain says : "According to the true view of contrariety, as given by De Morgan, the negative is a remainder, gained by the subtraction of the positive from the universe ; the negative of X is $U-X$, and may be symbolized by a distinct mark, x ; whence X and x are the opposites under a given universe ; not $-X$ is x, and not$-x$ is X."

Of the separation of logic and mathematic De Morgan says : "The effect has been unfortunate. . . . The sciences of which we speak may be considered either as disciplines of the mind, or as instruments in the investigation of nature and the advancement of the arts. In the former point of view their object is to strengthen the power of logical deduction by frequent examples ; to give a view of the difference between reasoning on probable premises and on certain ones by the construction of a body of results which in no case involve any of the uncertainty arising from the previous introduction of what may be false ; to establish confidence in abstract reasoning by the exhibition of processes whose results may be verified in many ways ; to help in enabling to acquire correct notions of generalization ; to give caution in receiving that which at first sight appears good reasoning ; to instill a correct estimate of the powers of the mind by pointing out the enormous extent of the consequences which may be developed out of a few of its most fundamental notions ; and to give the luxury of pursuing a study in which self-interest cannot lay down premises nor deduce conclusions.

As instruments in the investigation of nature and the advancement of the arts it is the object of these two sciences to find out truth in every matter in which nature is to be investigated, or her powers and those of the mind to be applied to the physical progress of the human race, or their advancement in the knowledge of the material creation."

De Morgan was fond of laughing at the metaphysicians. He says : "We know all about *can* and *cannot* from our cradles ; we never feel the same assurance about *is* and *is not.*

A philosopher, in a dark age, may determine to set out with a knowledge of the naturally possible and impossible ; but not even a philosopher ever pretended to set out with a knowledge of the existent and non-existent."

Aristotle and all the old logicians said that the whole of the middle term must be taken in at least one of the premises. As they put it, the middle term must be distributed at least once in the premises, otherwise the minor term may be compared with one part and the major with another part of it. From

Some men are poets,
Some men are Indians,

nothing follows. But the Aristotelians were wrong, as De Morgan clearly showed in his doctrine of Plurative Judgments. For example, if we have given the premises,

Most men are uneducated,
Most men are superstitious,

according to Aristotle we are not warranted in drawing any conclusion; for the middle term is men, and in neither premise is anything said about all men.

But, in point of fact, we can draw the perfectly valid conclusion,

Some uneducated men are superstitious.

Again Aristotle is contradicted by numerically definite judgments. In these there is inference when the quantities of the middle term *in the two premises together* exceed the whole quantity of that term.

Lambert first thought of this principle. De Morgan reconceived it and extended its use.

Suppose we grant the premises,

Two-thirds of all adults are women.

The number of women who have been married is never greater than the *total* number of men. It follows that half the entire number of women are single.

Still, easy and certain as such reasoning is, it may be difficult to a logician trained only in the traditional logic.

In a Princeton "Manual of Logic" the only numerically definite syllogism given was erroneous, and stood so for years. I stated this to the author, and in the latest stereotyped edition it has been changed. The Syllogism he gave was as follows:

"60 out of every 100 are unreflecting.
"60 out of every 100 are restless.
"Therefore, 20 out of every 100 restless persons are unreflecting."

After pointing out to him the fault in what he had been teaching for years, the following has been substituted:

"60 out of this 100 are unreflecting.
"60 out of this 100 are restless.
" ∴ 20 restless persons are unreflecting."

But De Morgan's greatest work was connected with his development of the Logic of Relatives, independently discovered by Robert Leslie Ellis after reading Boole's "Laws of Thought."

One of De Morgan's last memoirs, in the tenth volume of the "Cambridge Transactions," was on the Logic of Relations, which is, in the mathematical sense, a far-reaching generalization of the old logic. In our modern mathematics everything is generalized as far as possible. Every study of a generalization gives additional power over the particular. We need to go beyond and look back from an elevation.

Any first-rate mathematician working in logic would attempt to generalize, and, in fact, Boole generalized the scholastic logic in a manner entirely different from De Morgan. In De Morgan's view of the subject, the purely formal proposition with judgment wholly void of matter, is seen in "There is the probability x that X is in the relation L to Y." The syllogism is the determination of the relation which exists between two objects of thought by means of the relation in which each of them stands to some third object which is the middle term. The pure general form of the syllogism, when its premises are absolutely asserted, is as follows: X is in the relation L to Y, Y is in the relation M to Z; therefore X is in the relation "L of M," compounded of L and M, to Z. In ordinary logic the actual composition of the relation is made by our consciousness of its *transitive* character. A relation is transitive when, being compounded with itself, it reproduces itself; that is, L is transitive when every L of L is L; for example "brother."

Thus De Morgan broke away from that paltry narrowness which asserts that our minds in pure thinking can use nothing but the relation of identity; from the Jevons sophism that thought cannot move because all thought is the substitution of identicals.

So we see that in logic, as in mathematics, we may develop a whole system of theorems about symbols which are to be used in a given manner; and then to make this whole system true of a desired relation or subject matter we have only to show that this relation or subject matter fulfills the few fundamental principles of the system.

De Morgan treated of convertible and inconvertible relatives, repeating relatives; non-repeating relatives, transitive and intransitive relatives, and inaugurated a general system.

To what tremendous estate this system has grown may be seen in the 649 pages of Ernst Schroeder's Treatise, "Algebra und Logik der Relative;" Leipzig, Teubner, 1895, on whose first page, as founder of the system, stands the name of Augustus De Morgan.

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ON THE SOLUTION OF THE QUADRATIC EQUATION.

By G. A. MILLER, Ph. D., Paris, France.

One of the most important applications of substitution groups occurs in the theory of the solution of algebraic equations. It seems desirable that a fairly complete discussion of the solution of the quadratic equation should precede the study of this application. It is hoped that this discussion will not be without interest in itself even if the facts with which we have to deal are well known. As we shall need a clear idea of the *domain of rationality* we shall first develop several elementary concepts which involve the notion of groups and naturally lead to the more general concept of the domain of rationality.

Let us first consider the totality of numbers (T_1) formed by all the positive integers,* each positive integer occurring once and only once. By adding

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*We shall throughout confine our attention to the finite. Not only are the numbers to be regarded as finite but the number of times that a given operation is to be performed is also to be considered finite.